ON THE INTERPLAY BETWEEN BEHAVIORAL DYNAMICS AND SOCIAL INTERACTIONS IN HUMAN CROWDS

Citation for published version:
Bellomo, N, Gibelli, L & Outada, N 2019, 'ON THE INTERPLAY BETWEEN BEHAVIORAL DYNAMICS AND SOCIAL INTERACTIONS IN HUMAN CROWDS', *Kinetic and Related Models*, vol. 12, no. 2, pp. 397-409. https://doi.org/10.3934/krm.2019017

Digital Object Identifier (DOI):
10.3934/krm.2019017

Published In:
Kinetic and Related Models

Kinetic and Related Models
©American Institute of Mathematical Sciences
Volume **12**, Number **2**, April **2019**
doi:10.3934/krm.2019017
pp. **397–409**

ON THE INTERPLAY BETWEEN BEHAVIORAL DYNAMICS AND SOCIAL INTERACTIONS IN HUMAN CROWDS

Nicola Bellomo[(1)], Livio Gibelli[(2)] and Nisrine Outada[(3,4)]

[(1)] Politecnico of Torino and Collegio Carlo Alberto, Torino, Italy

[(2)] School of Engineering
University of Edinburgh, Edinburgh, United Kingdom

[(3)] Mathematics and Population Dynamics Laboratory-UMMISCO
Faculty of Sciences of Semlalia of Marrakech, Cadi Ayyad Univ., Morocco
[(4)] Jacques Louis-Lions Laboratory
Pierre et Marie Curie University, Paris 6, France

(Communicated by Axel Klar)

Abstract. This paper presents a computational modeling approach to the dynamics of human crowds, where social interactions can have an important influence on the behavioral dynamics of pedestrians. The modeling of the contagion and propagation of emotional states is carried out by looking at real physical situations where safety problems might arise in some specific circumstances. The approach is based on the methods of the kinetic theory of active particles. The evacuation of a metro station is simulated to enlighten the role of the emotional state in the overall dynamics.

1. **Introduction.**

The modeling, qualitative and computational analysis of human crowds is an interdisciplinary research field which involves a variety of analytic and numerical challenges, related not only to the derivation of models but also to their practical application to real-world problems.

The growing interest for this research field is motivated by the potential benefits for the society. As an example, the realistic modeling of human crowds can lead to simulation tools supporting crisis managers to handle emergency situations, as the sudden and rapid evacuation through complex venues. An interesting and challenging topic is the role played by the spreading of emotional states on the dynamics of the crowd, where stress induced by overcrowding may affect safety of the people [23, 24, 28, 30].

The existing literature on the mathematical modeling of human crowds is reported in some survey papers, which offer different viewpoints and modeling strategies in a field where a unified, commonly shared, approach does not exists yet. More in detail, the review [18] presents and critically analyzes the main features of the physics of a crowd viewed as a multi particle system and focuses on the modeling at the microscopic scale where it is assumed that pedestrians undergo individual based interactions. The survey [22] and the book [13] deal with the modeling at the macroscopic scale, by using methods analogous to those of hydrodynamics,

2010 *Mathematics Subject Classification.* Primary: 82D99, 91A15; Secondary: 91D10.
Key words and phrases. Crowd dynamics, living systems, complexity, kinetic theory, social dynamics, stress propagation.

where one of the most challenging conceptual difficulty consists in understanding how pedestrians in the crowd, viewed as a continuum, select their walking speed and velocity direction. References [6, 8] have proposed the concept of crowds as a living, hence complex, system. This approach requires the search of mathematical tools suitable to take into account, as far as it is possible, the complexity features of the system under consideration. Scaling problems and mathematical aspects are treated in the book [13], while the possible support that models can offer to crisis managers to handle emergency situations is discussed in the survey [7].

A critical analysis of the state of the art indicates that at least one issue has not been exhaustively addressed yet. *The greatest part of known models are based on the assumption of rational behaviors of individuals. However, in many real situations humans behave irrationally and this may lead to events which threaten their safety. Furthermore, when these conditions arise, small deviations in the input create large deviations in the output.*

Some of the topics mentioned in the above statement have been put in evidence in the review [30], where it is stressed that modeling approaches should be based on a careful understanding of human behaviors and that the majority of current crowd models do not yet effectively support crises managers. In addition, various researchers involved in the practical management of real crowd dynamics problems, including crisis and safety situations, have enlightened that social phenomena pervade heterogeneous crowds and can have an important influence on the interaction rules [7, 12, 19, 20, 21, 25, 26, 28, 30, 31]. Therefore it is of paramount importance to take into account both social and mechanical dynamics, as well as their complex interplay.

As it is well known, crowd dynamics models can be derived at three scales, namely *microscopic, macroscopic*, and the intermediate *mesoscopic* scale, which is occasionally referred to as *kinetic*. However none of them is fully satisfactory. One of the main issue is the need to properly account for the heterogeneous behavior of pedestrians and the subsequent multiple interactions. Models developed at the microscopic and macroscopic scales may adopt a mixture-like approach but, according to our bias, kinetic type models are more flexible. However, additional work is certainly needed to develop them towards the challenging objectives treated in this paper. Some introductory concepts have been proposed in the literature starting from Refs. [4, 6], where speed is related to an internal variable of a kinetic model suitable to describe stress conditions. More recently, Ref. [29] considers a dynamics in one space dimension described at the macroscopic scale, where panic is propagated by a BGK-like model [11]. A study on the role of social dynamics on individual interactions and their subsequent influence at the higher scale is carried out in Refs. [15, 16].

This paper is devoted to the modeling of the complex interplay between social and mechanical dynamics. Specifically we consider the propagation in time and space of stress conditions. The quality and the geometry of the venues where pedestrians move are also taken into account, as well as the interaction of pedestrians with obstacles and walls. The case study proposed in Section 3 shows that stress conditions can have huge impact on the overall dynamics of the crowd. In more detail, the so called *faster-is-slower* effect may take place, namely the individual speed increases in average but congested areas form which, in turn, lead to an increase of the evacuation time [24, 28, 30].

Following this preliminary introduction, the contents of this paper is presented as follows. Section 2 deals with the modeling of a crowd composed of individuals gathered in groups based on possibly different walking strategy and/or specific interactions' rules with other individuals. This section transfers into a mathematical framework the general concepts of human crowds, with the aim of providing the conceptual basis for the derivation of models which can be obtained by modeling the interactions at the micro-scale. Section 3 derives a specific model which has the capability of describing the spreading of the stressful conditions in crowds. Simulations are carried out for the evacuation from a platform of a metro station by solving the model through a Monte Carlo particle method [2, 3, 27]. Specific features of the patterns of the flow are enlightened with special emphasis on the propagation of pedestrians' emotional state through the crowd. Section 4 presents a critical analysis of the contents of the paper as well as an overview of research perspectives which are mainly focused on multiscale problems and on the conceptual links of models at each scale.

2. On a kinetic mathematical theory of social crowds.

This section deals with the first step towards the derivation of models by suitable developments of the kinetic theory for active particles [5]. It consists in formulating a general mathematical structure capable of describing heterogeneous human crowds in domains with complex geometries and taking into account the features of social dynamics which may affect the overall behavior of the crowd.

According to this theory, pedestrians are regarded as *active particles*, for short a-particles, whose state is identified, in addition to mechanical variables, such as position and velocity, by an additional variable which describes their emotional or social state, called *activity*. Pedestrians are then subdivided into *functional subsystems*, for short FSs, which group a-particles that share the same activity and/or mechanical purposes, although heterogeneously distributed within each FS.

The sequential steps of the modeling strategy are as follows:

(i) Selection of the social phenomena to be inserted in the model.

(ii) Selection of the modeling scale and derivation of a general mathematical structure consistent with the selected social dynamics.

(iii) Derivation of the model by inserting, into the said structure, the mathematical description of both mechanical and social interactions including their possible interplay.

It is clear that, no matter how general is the mathematical structure mentioned in Item (ii) models cannot be derived which account for whole variety of social phenomena. Therefore, in the following, only a specific case study is selected, namely the propagation of the emotional state through a crowd otherwise homogeneous.

2.1. Mechanical-social dynamics and representation.

Let us now provide a detailed description of the specific features that our paper takes into account in the search of a general mathematical structure which permits one to derive models of social crowds.

- The crowd is subdivided into groups labeled by the subscript $i = 1, \ldots, n$, corresponding to different FSs which comprise a-particles that share the same activity and/or mechanical purposes, for instance, heading for the same meeting point.

- The mechanical state of the a-particles is defined by position $\boldsymbol{x}$, velocity $\boldsymbol{v}$, while their emotional state is modeled by a variable, which is referred to as activity, which is assumed to take values in the domain $[0, 1]$.
- Interactions are supposed not only to modify the mechanical variables, but also the activity which, in turn, may affect the mechanical dynamics. Indeed, different behaviors induce different interactions and, in turn, different pedestrians' trajectories.

The *microscopic state* of the a-particles, is defined by position $\boldsymbol{x}$, velocity $\boldsymbol{v}$, and activity u. Dynamics in two space dimensions is considered, and polar coordinates are used for the velocity variable, namely $\boldsymbol{v} = \{v, \theta\}$, where v is the speed and θ denotes the velocity direction. Dimensionless, or normalized, quantities are used by referring the components of $\boldsymbol{x}$ to a characteristic length ℓ, while the velocity modulus is divided by the limit speed V_ℓ, which is the speed that can be reached by a fast pedestrian in free flow conditions; t is the dimensionless time variable obtained referring the real time to a suitable characteristic time T_c identified by the ratio between ℓ and V_ℓ. The limit velocity depends on the quality of the environment, such as presence of positive or negative slopes, lighting and so on. The lower the quality of the venue, the smaller is the limit velocity.

The *mesoscopic (kinetic) representation* of each FS at time t is delivered by the probability distribution function over the microscopic state of pedestrians:

$$f_i = f_i(t, \boldsymbol{x}, v, \theta, u), \quad \boldsymbol{x} \in \Sigma \subset \mathbb{R}^2, \quad v \in [0, 1], \quad \theta \in [0, 2\pi) \quad u \in [0, 1]. \tag{1}$$

If f_i is locally integrable then $f_i(t, \boldsymbol{x}, \boldsymbol{v}, u)\, d\boldsymbol{x}\, d\boldsymbol{v}\, du$ is the (expected) infinitesimal number of pedestrians of the i-th FS whose micro-state, at time t, is comprised in the elementary volume $[\boldsymbol{x}, \boldsymbol{x} + d\boldsymbol{x}] \times [\boldsymbol{v}, \boldsymbol{v} + d\boldsymbol{v}] \times [u, u + du]$ of the space of the micro-states, corresponding to the variables space, velocity and activity. The statistical distributions f_i are divided by n_{M}, which defines the maximal full packing density of pedestrians and it is assumed to be approximately seven pedestrians per square meter. *Macroscopic observable quantities* can be obtained, under suitable integrability assumptions, by weighted moments of the distribution functions.

2.2. **Derivation of a mathematical structure.** Interactions lead to a decision process by which each a-particle modifies its activity and adopts a walking strategy depending on the micro-states and distribution functions of the neighboring particles in its interaction domain Ω_s. In principle, Ω_s is a circular sector, with radius R_s, symmetric with respect to the velocity direction being defined by the visibility angles Θ and $-\Theta$. Note that the interaction domain can be geometrically reduced in presence of obstacles and walls and interactions depend on the perceived density. Note that, the density perceived by a-particles is higher or lower than the real one depending on whether the density gradient in their velocity direction is positive or negative.

The derivation of a general mathematical structure is based on the assumption that interactions involve, at each time t and for each FS, three types of a-particles: The *test particle*, the *field particle*, and the *candidate particle*. Their distribution functions are $f_i(t, \boldsymbol{x}, \boldsymbol{v}, u)$, $f_k(t, \boldsymbol{x}, \boldsymbol{v}^*, u^*)$, and $f_h(t, \boldsymbol{x}, \boldsymbol{v}_*, u_*)$, respectively. The test particle, is representative, for each FS, of the whole system, while the candidate is the generic field particle which can acquire, in probability, the micro-state of the test particle after interaction with the field particle. The test particle loses its state by interaction with the field particle.

Based on this assumption, the modeling of interactions requires to specify the *interaction rate*, η, and the *transition probability density*, $\mathcal{A}$. These quantities depend on the micro-state and on the distribution function of the interacting particles, as well as on the quality of the environment where the crowd moves. The definition of these two quantities is reported in the following, where the term i-particle is used to denote a-particles belonging to the i-th FS.

- *Interaction rate:* Candidate (or test) h-particle in $\boldsymbol{x}$ interact with a field k-particles in Ω_s with a frequency denoted by $\eta_{hk}[\mathbf{f}](\boldsymbol{x}, \boldsymbol{v}_*, \boldsymbol{v}^*, u_*, u^*; \alpha)$, where square brackets denote the functional dependence on the whole set of distribution functions, $\mathbf{f} = \{f_i\}$. Note that the parameter α is introduced to quantify the local quality of the environment, namely $\alpha = \alpha(\boldsymbol{x}) \in [0, 1]$, where $\alpha = 0$ corresponds the worse conditions which prevent motion, while $\alpha = 1$ corresponds to the best ones, which allows a rapid motion.
- *Transition probability density:* The probability density that a candidate i-particle in $\boldsymbol{x}$ with state $\{\boldsymbol{v}_*, u_*\}$ shifts to the state of the i-test particle due to the interaction with a field k-particle in Ω_s with state $\{\boldsymbol{v}^*, u^*\}$ is denoted by $\mathcal{A}_{ik}[\mathbf{f}](\boldsymbol{v}_* \to \boldsymbol{v}, u_* \to u | \boldsymbol{x}, \boldsymbol{v}_*, \boldsymbol{v}^*, u_*, u^*; \alpha)$.

A general mathematical structure can then be derived by a balance of the number of a-particles in the elementary volume of the space of the micro-states. This conservation equation is obtained by equating the variation rate of the number of a-particles plus the transport due to the velocity variable to net flux rates within the same FS and across FSs. The result is a system of integro-differential equations describing the time dynamics of the distribution functions f_i.

We can take advantage of a general structure already available in the literature, see Chapter 3 of [5], which is here reported for sake of completeness:

$$
\begin{aligned}
(\partial_t + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}})\, f_i(t, \boldsymbol{x}, \boldsymbol{v}, u) = \\
= \sum_{k=1}^{n} \int_{D^2} \eta_{ik}[\mathbf{f}](\boldsymbol{x}, \boldsymbol{v}_*, \boldsymbol{v}^*, u_*, u^*; \alpha)\, \mathcal{A}_{ik}[\mathbf{f}](\boldsymbol{v}_* \to \boldsymbol{v}, u_* \to u | \boldsymbol{x}, \boldsymbol{v}_*, \boldsymbol{v}^*, u_*, u^*; \alpha) \\
\times\; f_i(t, \boldsymbol{x}, \boldsymbol{v}_*, u_*) f_k(t, \boldsymbol{x}, \boldsymbol{v}^*, u^*)\, d\boldsymbol{v}_*\, d\boldsymbol{v}^*\, du_*\, du^* \\
- f_i(t, \boldsymbol{x}, \boldsymbol{v}, u) \sum_{k=1}^{n} \int_{D} \eta_{ik}[\mathbf{f}](\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{v}^*, u, u^*; \alpha)\, f_k(t, \boldsymbol{x}, \boldsymbol{v}^*, u^*)\, d\boldsymbol{v}^*\, du^*, \qquad (2)
\end{aligned}
$$

where $D = [0, 1] \times [0, 2\pi) \times [0, 1]$ the terms η_{ik} and $\mathcal{A}_{ik}$ have been defined above.

This structure permits one to take into account the presence of different functional subsystems, for example pedestrians moving towards different directions, presence of leaders and so on. This structure has been applied to model the dynamics of a variety of systems of self-propelled particles [5]. We do not naively claim that it is the only mathematical structure capable of describing living systems and we refer the interested reader to Chapters 3 and 4 of the book [5] for a detailed discussion of its upsides and downsides.

3. **From the mathematical structure to modeling.** The mathematical structure presented in the preceding section provides the conceptual framework for the derivation of models which can be obtained by selecting the FS relevant to the specific study to be developed and by modeling interactions related to the strategies developed by a-particles within each FS.

This section shows how a specific model, of interest for the applications, can be derived. In more detail, we look for a model which permits one to understand how the stress propagates through a crowd and modify pedestrians' flow characteristics. The derivation of the model is discussed in the next subsection, while in the subsequent subsection some sample simulations are reported which depict the aforementioned propagation dynamics in time and space.

3.1. **Dynamics with stress propagation.** Let us consider a crowd in a venue Σ and let us assume that all pedestrians can be grouped in only one FS. Accordingly, the mathematical structure used towards the modeling is given by Eq. (2) specialized to a system whose state is described by the distribution function $f = f(t, \boldsymbol{x}, \boldsymbol{v}, u)$. Furthermore, the various terms are defined as follows.

Interaction rate: A simple assumption consists in supposing that it grows with the activity variable and with the density perceived by pedestrians starting from a minimal value η_0, namely $\eta[f] = \eta_0(1 + \beta\, u\, \rho_p[f])$, where β is a positive defined constant and ρ_p the perceived density (see Eq. (13)). A minimal model is obtained with $\eta \cong \eta_0$.

Transition probability density: Following Ref. [8], interactions are supposed to trigger a decision process which comprises the following steps: (1) Exchange of the stress state; (2) Selection of the walking direction; (3) Selection of the walking speed. Decisions are supposed to be sequential and dependent on the local flow conditions. Hence, the transition probability density factorizes:

$$\mathcal{A}[\mathbf{f}](\boldsymbol{v}_* \to \boldsymbol{v}, u_* \to u) = \mathcal{A}^u[\mathbf{f}](u_* \to u) \times \mathcal{A}^\theta[\mathbf{f}](\theta_* \to \theta) \times \mathcal{A}^v[\mathbf{f}](v_* \to v). \tag{3}$$

By further assuming that the output of the interaction is a delta function over the most probable state, the following model can be obtained:

1. Dynamics of the emotional state: The stress is supposed to spread through the crowd based on the transition probability density:

$$u^* > u_* : \quad \mathcal{A}^u(u_* \to u | u_*, u^*) = \delta\big(u - \varepsilon(u^* - u_*)(1 - u_*)\big), \tag{4}$$

and

$$u^* \leq u_* : \quad \mathcal{A}^u(u_* \to u | u_*, u^*) = \delta\big(u - u_*\big). \tag{5}$$

where ε is a parameter that measures the tendency of pedestrians to modify their emotional state.

2. Dynamics of the velocity direction: It is expected that at high density, pedestrians walk away from overcrowded areas by moving in the direction of $\boldsymbol{\nu}_V$ (direction of the less congestion), while at low density, pedestrians head for the target identified $\boldsymbol{\nu}_T$ (as an example exit doors) unless their level of stress is high in which case they tend to follow the mean stream as given by $\boldsymbol{\nu}_S$ (direction of the stream). However, the presence of walls also affect the decision process by which pedestrians select their preferred walking direction. Accordingly, it is assumed that the preferred walking direction θ is chosen in two steps. Firstly, a walking direction θ_1 is defined by weighting the directions $\boldsymbol{\nu}_V$, $\boldsymbol{\nu}_T$, and $\boldsymbol{\nu}_S$, being the weights the density and the level of stressful conditions. If this walking direction points towards an exit area, then it is not modified. Otherwise, the preferred walking direction is defined by a weighted choice between θ_1 and the direction θ_T of $\boldsymbol{\nu}_T$, where the weight is given by the distance d_w from the closest wall. Accordingly, the transition probability density for the angles is defined as follows:

$$\mathcal{A}^\theta[\rho, \boldsymbol{x}, \boldsymbol{\xi}](\theta_* \to \theta) = \delta\left(\theta - \theta_*\right), \quad \text{with} \quad \theta = (1 - d_w)\theta_T + d_w \theta_1, \tag{6}$$

where d_w is assumed to be equal to one if θ_1 is directed towards an exit area and θ_1 is the direction of the unit vector $\boldsymbol{\nu}_1$ given by:

$$\boldsymbol{\nu}_1 = \frac{\rho\boldsymbol{\nu}_V + (1-\rho)\,\dfrac{u\boldsymbol{\nu}_S + (1-u)\boldsymbol{\nu}_T}{\|u\boldsymbol{\nu}_S + (1-u)\boldsymbol{\nu}_T\|}}{\left\|\rho\boldsymbol{\nu}_V + (1-\rho)\,\dfrac{u\boldsymbol{\nu}_S + (1-u)\boldsymbol{\nu}_T}{\|u\boldsymbol{\nu}_S + (1-u)\boldsymbol{\nu}_T\|}\right\|}\,. \tag{7}$$

The unit vectors $\boldsymbol{\nu}_V$ and $\boldsymbol{\nu}_S$, in particular, are given by:

$$\boldsymbol{\nu}_V = -\frac{\nabla_{\boldsymbol{x}}\rho}{\|\nabla_{\boldsymbol{x}}\rho\|}\,, \qquad\qquad \boldsymbol{\nu}_S = \frac{\boldsymbol{\xi}}{\|\boldsymbol{\xi}\|}\,, \tag{8}$$

where ρ and $\boldsymbol{\xi}$ are the pedestrian density and mean velocity, respectively.

3. Dynamics of the speed: Once the direction of motion has been selected, the walker adjusts the speed to the local density and mean speed conditions. A specific model, in agreement with [9] can be used:
If $\xi \geq v_*$:

$$\mathcal{A}^v(v_* \to v)[\rho,\xi] = p_a(\alpha,u,\rho)\,\delta(v - \xi_a(\alpha,u,\rho)) + (1 - p_a(\alpha,u,\rho))\,\delta(v - v_*), \tag{9}$$

and, if $\xi < v_*$:

$$\mathcal{A}^v(v_* \to v)[\rho,\xi] = p_d(\alpha,u,\rho)\delta(v - \xi_d(\xi,\rho_p)) + (1 - p_d(\alpha,u,\rho))\,\delta(v - v_*), \tag{10}$$

where

$$p_a(\alpha,u,\rho) = \alpha\, u(1-\rho_p), \qquad \xi_a(\alpha,u,\rho) = \xi + \alpha\, u(1-\rho_p)(\alpha u - \xi), \tag{11}$$

and

$$p_d(\alpha,u,\rho) = (1-\alpha\, u)\rho_p, \qquad \xi_d(u,\rho) = \xi(1-\rho_p). \tag{12}$$

In Eqs. (11) and (12), ρ_p denotes the perceived density. The following model has been proposed in [8]:

$$\rho_p = \rho_p[\rho] = \rho + \frac{\partial_1\rho}{\sqrt{1+(\partial_1\rho)^2}}\,\left[(1-\rho)\,H(\partial_1\rho) + \rho\,H(-\partial_1\rho)\right]\,, \tag{13}$$

where ∂_1 denotes the derivative along the direction θ_1 while $H(\cdot)$ is the Heaviside function $H(\cdot \geq 0) = 1$, and $H(\cdot < 0) = 0$. Accordingly, a particle moving in the direction θ_1 perceives a density $\rho_p > \rho$ when the density increases, while $\rho_p < \rho$, when the density decreases. Note that the density ρ_p provided by Eq. (13) takes value in the range $[0,1]$.

This heuristic model of speed dynamics is based on the following rationale: *If the walkers speed is lower (higher) than the mean speed, then the walker tends to increase (decrease) the speed according to a decision process which is enhanced by low values of the perceived density and by the goodness of the venue's quality.*

It is worth noticing that, although the walking strategy encompasses the tendency to keep distance from walls, some pedestrians, in probability, might reach boundaries and, therefore, an appropriate scattering model needs to be given. In more detail, it is supposed that at the walls pedestrians keep their speed constant but modify the velocity direction, making it tangent to the wall.

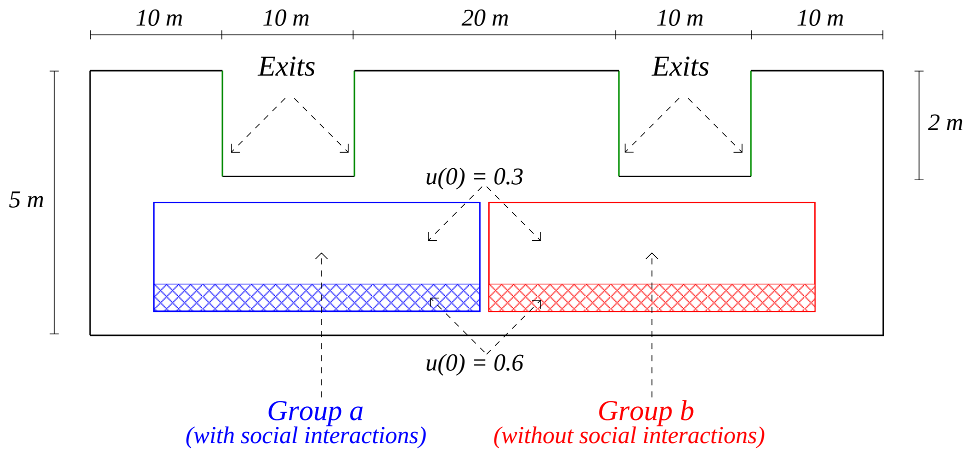


FIGURE 1. Geometry of the venue

3.2. **A case study and simulations.** In order to assess the predictive ability of the kinetic model described in the previous section, sample simulations are carried out which aim at enlightening how social interactions may affect the dynamics of pedestrians evacuating from a metro platform. The geometry of the venue and the initial conditions of the crowd are depicted in Fig. 1. The platform has dimension $60m \times 5m$. At time $t = 0\,s$, a uniform crowd of 50 people stand at rest uniformly distributed in a rectangular region of dimension $50m \times 2m$. The crowd is supposed to be constituted of two groups. Pedestrians of group a (blue color) are located on the left half of the platform, $x < 30$, and it is assumed that they interact socially, $\varepsilon = \varepsilon_a = 10^{-2}$. Instead, pedestrians of group b (red color) are located on the right half of the platform, $x > 30$, and, unlike pedestrians of group a, it is assumed that they keep their emotional state constant during the simulation, $\varepsilon = \varepsilon_b = 0$. Finally, pedestrians closest to the exits are supposed to have initially a level of stress lower than the one of those who are further away, namely $u(0) = u_l = 0.3$ (patterned blue and red areas in Fig. 1) and $u(0) = u_h = 0.6$, respectively.

The kinetic model has been solved by a stochastic particle scheme which closely resembles the Direct Simulation Monte Carlo (DSMC) method. The distribution function is represented by a collection of about $5 \cdot 10^5$ computational particles whose positions and velocities evolve in time by a sequence of time steps, each consisting of a free flight and a local collision sub-steps. The former corresponds to the streaming operator on left hand-side of Eq. (2), whereas the latter is performed according to stochastic rules, consistent with the structure of the collision terms on the right-hand side of Eq. (2) and the transition probability densities given by Eqs. (4),(5),(6),(9), and (10). The space domain to be simulated is covered by a mesh of cells. These cells are used to collect together particles that may collide and also for the sampling of macroscopic properties such as density and mean velocity.

Figures 2-3 show the contour plots at different times of the mean density of the emotional state. The latter quantity is defined as

$$\rho(t, \boldsymbol{x})\bar{u}(t, \boldsymbol{x}) = \int f(t, \boldsymbol{x}, \boldsymbol{v}, u) u \, d\boldsymbol{v} \, du. \tag{14}$$

As the simulation starts, the crowd splits with pedestrians of each group heading for the closest exit.

Figures 2 and 3 show contour plots of the mean density of the emotional state at initial times and at a later stage, respectively. It is apparent that stressful conditions rapidly spread through the group a (darker colors refer to higher values of the mean density of the emotional state), and significantly affect the evacuation time of group a compared to the one of group b in which the social interactions are disregarded. Note that pedestrians with higher level of stress have the tendency to walk with an higher speed, however they undergo high crowd concentration.

The spreading is clearly shown in Fig. 4 which reports the averaged value of the emotional state over the whole domain, $\bar{U}$, for different values of the social interaction parameter, ε. When $\varepsilon = 0$, as the case of group b, $\bar{U}$ is first constant and then monotonically decreases when pedestrians with the higher level of stress exit the domain.

By contrast, for $\varepsilon = 10^{-1}$, $\bar{U}$ monotonically increases because, due to social interactions, pedestrians with the higher level of stress affect the whole group a before leaving the domain. Accordingly, the group a becomes homogeneous as far as the level of stress is concerned and $\bar{U}$ reaches its maximum value, namely u_h.

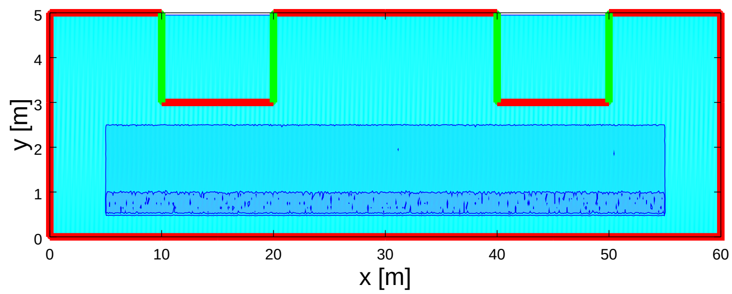
The intermediate behavior of $\bar{U}$ is observed for $\varepsilon = 10^{-2}$ which is not unexpected. Indeed, the lower ε, the larger is the time needed for the stress to spread through the group a. Accordingly, some pedestrians with the higher level of stress may leave the domain without significantly interacting with the others and this leads to the non-monotonic behavior of $\bar{U}$.

4. **Critical analysis and research perspectives.** In this paper, a kinetic theory approach has been proposed to the modeling of crowd dynamics in presence of social phenomena which can modify the rules of mechanical interactions. A specific social dynamics have been studied, namely the propagation of stress conditions. The case study proposed in Section 3 has shown that their spreading significantly affect the overall dynamics and the density patterns of the crowd, leading to overcrowding in some areas.

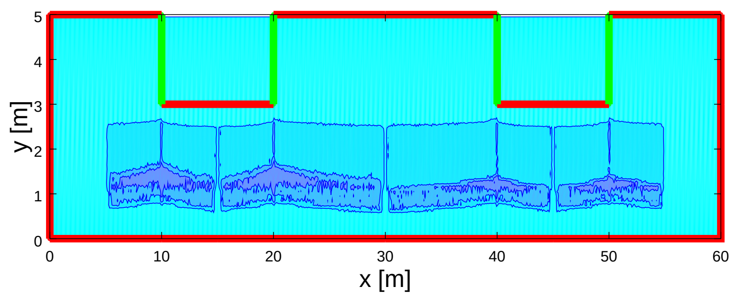
The achievements presented in the preceding sections motivate further studies to broaden the variety of social phenomena that can be potentially included in the proposed modeling approach. Examples range from the dynamics under the action of leaders to the description of extreme situations, where antagonist groups contrast each other in a crowd and the violence possibly propagates due to social interactions [10, 17]. These developments can be definitely inserted into a possible research program which is strongly motivated by the security problems of our society. The survey [1] provides a relevant contribution to modeling specific social phenomena.

By returning to the scaling problem mentioned in Sections 1 and 2, it is worth pointing out that it would be very important to account for social behaviors also in the modeling at the microscopic and macroscopic scales. A detailed study can be then carried out to assess upsides and downsides of each modeling scale.

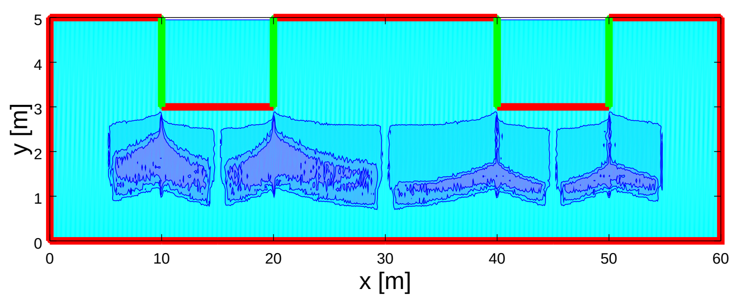
This type of analysis should not hide the conceptual link which joins the different scales. In fact, the modeling of individual based interactions (microscopic scale) should be used to derive kinetic type models (mesoscopic scale), while hydrodynamic models (macroscopic scale) should follow from the kinetic theory description by using asymptotic methods where a small parameter corresponding to the distances between individuals is let to tend to zero. By contrast, models are often



(A) $t = 0\,s$.



(B) $t = 1\,s$.

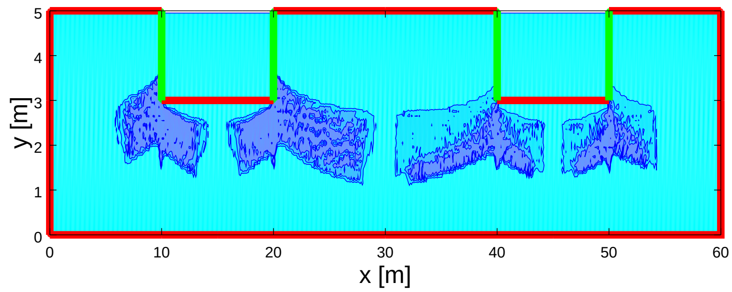


(C) $t = 2\,s$.

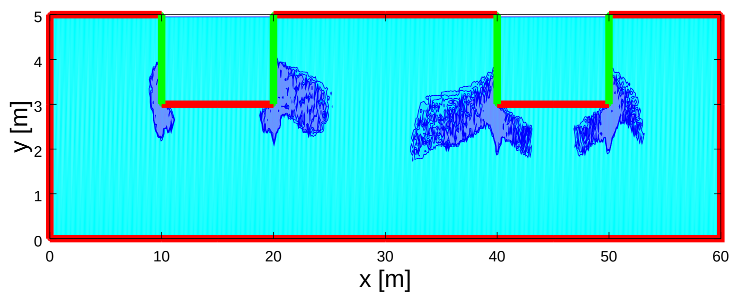
FIGURE 2. Density contour plots of the mean density of the emotional state, $\rho\bar{u}$, with (pedestrians on the left) and without (pedestrians on the right) social interactions at different times.

derived independently at each scale, which prevents a real multiscale approach. Let us point out that including the emotional states in a crowd is not simply a matter of introducing additional parameters, but it requires to add new variables in the pedestrians' microscopic state. This means to add new terms in the vector differential system which describes the crowd dynamics at the microscopic scale and additional interactions and moments in the kinetic theory approach.

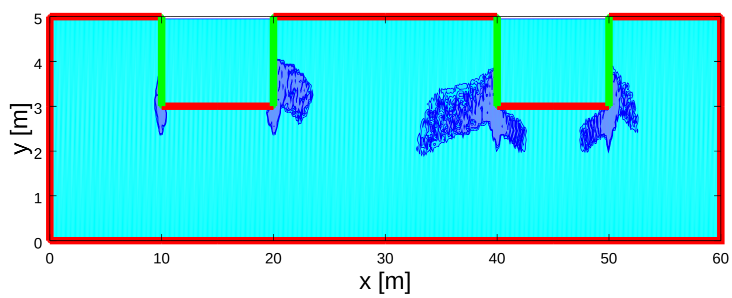
Some achievements have already been obtained on the derivation of macroscopic equations from the kinetic type description for crowds in unbounded domains [4]. Further results, definitely worth to be mentioned, are those presented in Ref. [14],



(A) $t = 5\, s$.



(B) $t = 15\, s$.



(C) $t = 20\, s$.

FIGURE 3. Density contour plots of the mean density of the emotional state, $\rho\bar{u}$, with (pedestrians on the left) and without (pedestrians on the right) social interactions at different times.

where the authors derive, by means of a mean field approximation, macroscopic models from individual based models. However, the problem of how the structure of macroscopic models is modified by social behaviors is still poorly addressed in the literature.

Hence a challenging objective is the development of a system approach to human crowds, where models derived at the three different scales are coupled to describe the pedestrians' dynamics in complex venues in presence of areas of low and high density. This objective can only be achieved by deriving models at the microscopic scale

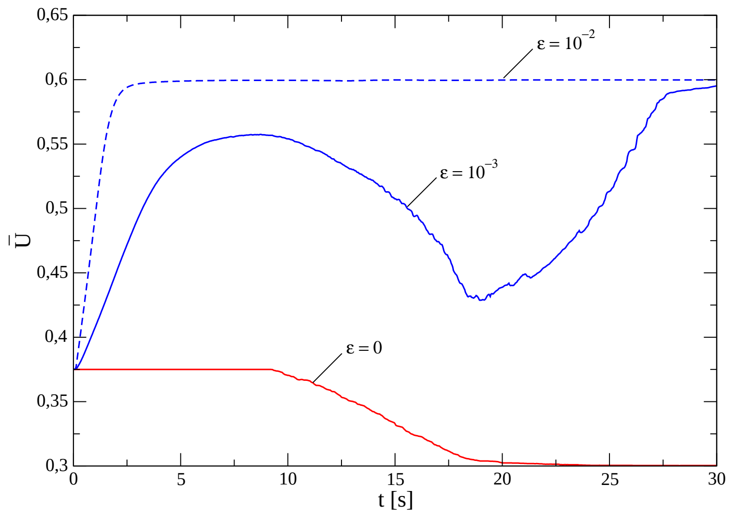


FIGURE 4. Averaged value of the emotional state of the crowd, $\bar{U}$, versus time for different values of the social parameter.

consistent with models at the macroscopic scale, being the intermediate description offered by the kinetic theory approach.

REFERENCES

[1] G. Ajmone Marsan, N. Bellomo and L. Gibelli, Stochastic evolutionary differential games toward a systems theory of behavioral social dynamics, *Math. Models Methods Appl. Sci.*, **26** (2016), 1051–1093.

[2] V.V. Aristov, *Direct Methods for Solving the Boltzmann Equation and Study of Nonequilibrium Flows,* Springer-Verlag, New York, 2001.

[3] P. Barbante, A. Frezzotti and L. Gibelli, A kinetic theory description of liquid menisci at the microscale, *Kinet. Relat. Models*, **8** (2015), 235–254.

[4] N. Bellomo and A. Bellouquid, On multiscale models of pedestrian crowds from mesoscopic to macroscopic, *Comm. Math. Sciences*, **13** (2015), 1649–1664.

[5] N. Bellomo, A. Bellouquid, L. Gibelli and N. Outada, *A Quest Towards a Mathematical Theory of Living Systems,* Birkhäuser, New York, 2017.

[6] N. Bellomo, A. Bellouquid, and D. Knopoff, From the micro-scale to collective crowd dynamics, *Multiscale Model. Sim.*, **11** (2013), 943–963.

[7] N. Bellomo, D. Clark, L. Gibelli, P. Townsend, and B.J. Vreugdenhil, Human behaviours in evacuation crowd dynamics: From modelling to ig data toward crisis management. *Phys. Life Rev.*, **18** (2016), 1–21.

[8] N. Bellomo and L. Gibelli, Toward a behavioral-social dynamics of pedestrian crowds, *Math. Models Methods Appl. Sci.*, **25** (2015), 395–400.

[9] N. Bellomo and L. Gibelli, Behavioral crowds: Modeling and Monte Carlo simulations toward validation, *Comp. & Fluids*, **141** (2016), 13–21.

[10] D. Burini, S. De Lillo and L. Gibelli, Stochastic differential "nonlinear" games modeling collective learning dynamics, *Phys. Life Rev.*, **16** (2016), 123–139.

[11] C. Cercignani, R. Illner and M. Pulvirenti, *The Mathematical Theory of Diluted Gas,* Springer, Heidelberg, New York, 1994.

[12] A. Corbetta, A. Mountean and K. Vafayi, Parameter estimation of social forces in pedestrian dynamics models via probabilistic method, *Math. Biosci. Eng.*, **12** (2015), 337–356.

[13] E. Cristiani, B. Piccoli and A. Tosin, *Multiscale Modeling of Pedestrian Dynamics,* Springer, 2014.

[14] P. Degond, C. Appert-Rolland, J. Pettré and G. Theraulaz, Vision based macroscopic pedestrian models, *Kinetic Related Models*, **6** (2013), 809–839.

[15] P. Degond, C. Appert-Rolland, M. Moussaïd, J. Pettré and G. Theraulaz, A hierarchy of heuristic-based models of crowd dynamics, *J. Stat. Phys.*, **152** (2013), 1033–1068.
[16] P. Degond, J.-G. Liu, S. Merino-Aceituno and T. Tardiveau, Continuum dynamics of the intention field under weakly cohesive social interaction, *Math. Models Methods Appl. Sci.*, **27** (2017), 159–182.
[17] J. M. Epstein, Modeling civil violence: An agent based computational approach, *Proc. Nat. Acad. Sci.*, **99** (2002), 7243–7250.
[18] D. Helbing, Traffic and related self-driven many-particle systems, *Rev. Modern Phys.*, **73** (2001), 1067–1141.
[19] D. Helbing, I. Farkas and T. Vicsek, Simulating dynamical feature of escape panic. *Nature*, **407** (2000), 487–490.
[20] D. Helbing and A. Johansson, Pedestrian crowd and evacuation dynamics, *Enciclopedia of Complexity and System Science*, (2009), 6476–6495.
[21] D. Helbing, A. Johansson and H. Z. Al-Abideen, Dynamics of crowd disasters: An empirical study, *Phys. Rev. E*, **75** (2007), 046109.
[22] R. L. Hughes, The flow of human crowds, *Annu. Rev. Fluid Mech.*, **35** (2003), 169–182.
[23] M. Kinateder *et al.*, Human behaviour in severe tunnel accidents: Effects of information and behavioural training, *Transp. Res. Part F: Traffic Psychology and Behaviour*, **17** (2013), 20–32.
[24] J. Lin and T. A. Luckas, A particle swarm optimization model of emergency airplane evacuation with emotion, *Net. Het. Media*, **10** (2015), 631–646.
[25] M. Moussaïd, D. Helbing, S. Garnier, A. Johansson, M. Combe and G. Theraulaz, Experimental study of the behavioural mechanisms underlying self-organization in human crowds, *Proc. Roy. Soc. B*, **276** (2009), 2755–2762.
[26] M. Moussaïd and G. Theraulaz, Comment les piétons marchent dans la foule. *La Recherche*, **450** (2011), 56–59.
[27] L. Pareschi and G. Toscani, *Interacting Multiagent Systems: Kinetic Equations and Monte Carlo Methods*, Oxford University Press, Oxford, 2014.
[28] F. Ronchi, F. Nieto Uriz, X. Criel and P. Reilly, Modelling large-scale evacuation of music festival. *Fire Safety*, **5** (2016), 11–19.
[29] L. Wang, M. Short and A. L. Bertozzi, Efficient numerical methods for multiscale crowd dynamics with emotional contagion, *Math. Models Methods Appl. Sci.*, **27** (2017), 205–230.
[30] N. Wijermans, C. Conrado, M. van Steen, C. Martella and J. L. Li, A landscape of crowd management support: An integrative approach, *Safety Science*, **86** (2016), 142–164.
[31] N. Wijermans, R. Jorna, W. Jager, T. van Vliet and O. M. J. Adang, CROSS: Modelling crowd behaviour with social-cognitive agents, *Journal of Artificial Societies and Social Simulation*, **16** (2013), 1–14.

Received for publication May 2018.

E-mail address: nicola.bellomo@polito.it
E-mail address: Livio.Gibelli@ed.ac.uk
E-mail address: aoutada1991@gmail.com